Numerically quantified expressions

Chris Cummins
Napoleon Katsos

Research Centre for English and Applied Linguistics,
University of Cambridge

Outline

- Numerically quantified expressions
  - and their analysis, classical or otherwise
- New data on numerical quantifier interpretation
  - and its theoretical implications
- Towards a unified account of numerical quantifier meaning?

Comparative and superlative quantifiers

- Comparative: "**more** than", "**less**/**fewer** than"

- Superlative: "at **least**", "at **most**"

Inter-definable?

More than	Fewer than	At least	At most
>	<	≥	≤

- "x is more than y" => x > y
  => x ≥ (y+1) => "x is at least (y+1)"

- "x is less than y" => x < y
  => x ≤ (y-1) => "x is at most (y-1)"

- Not true on **Q**, **R** (no infinitesimals), therefore not true for measurements etc.

Not inter-definable?

More than	Fewer than	At least	At most
>	<	Certainly n exist; possibly more	Possibly n exist; certainly not more

(Geurts and Nouwen 2007)

(Narrow columns correspond to narrow semantics)

Why not interdefinable?

- If not, then redundancy throughout the system

- Distribution of superlative quantifiers suggests modality

  - *[?]not at most three people
    (cf. *[?]not maybe three people)

- Experimental evidence…

Predictions from modality

- Superlative quantifiers are modal, comparative quantifiers are not =>

    - Superlative quantifiers will give rise to processing delays

    - Superlative quantifiers will be more difficult to acquire, consequently mastered later

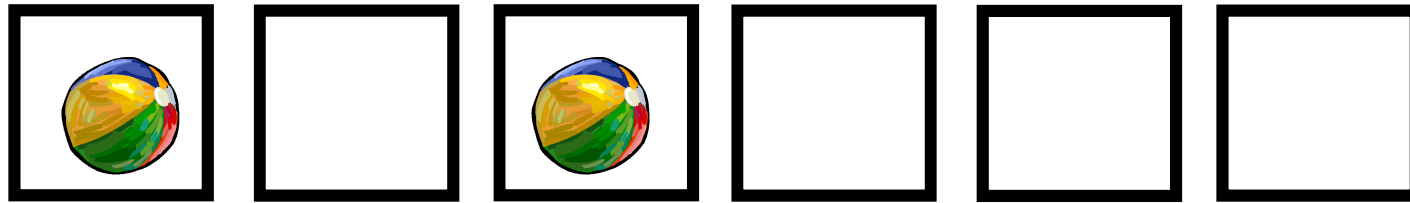   - Both types of quantifiers give rise to different patterns of reasoning

Evidence

- Musolino (2004), Geurts et al. (in press): later acquisition of superlative quantifiers

- Geurts et al. (in press): Slower processing of superlative quantifiers
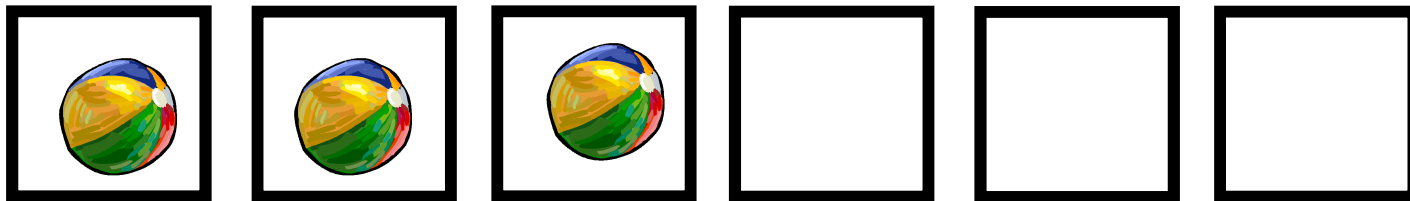
Order of acquisition

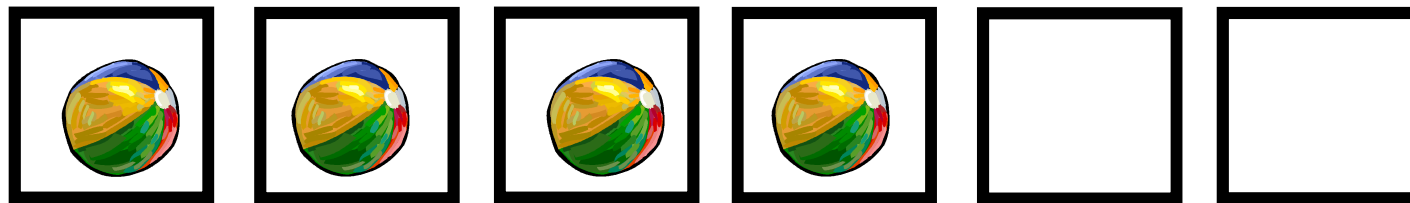"Please make the toys and boxes match my
    sentence..."
"At least three of the boxes have a ball"

Complexity of superlatives?

More than	Fewer than	At least	At most
>	<	≥	≤
"Greater than"	"Less than"	"Greater than OR equal to"	"Less than OR equal to"

Predictions from modality

- Superlative quantifiers are modal, whereas comparative quantifiers are not =>

    - Superlative quantifiers will give rise to processing delays

    - Superlative quantifiers will be more difficult to acquire, consequently mastered later

    - Both types of quantifiers give rise to different patterns of reasoning

Predictions from complexity

- Superlative quantifiers are NOT modal, just as comparative quantifiers are not !=>

  - Superlative quantifiers will give rise to processing delays

  - Superlative quantifiers will be more difficult to acquire, consequently mastered later

  - Both types of quantifiers give rise to different patterns of reasoning

Logical relatedness (G&N)

Q. Is it the case that the first of these two sentences implies the second?

Berta had at most 2 drinks

Berta had at most 3 drinks

At most 2 => At most 3: 13% (or 2%) acceptance

Fewer than 3 => Fewer than 4: 71% (or 63%)

Logical relatedness (G&N)

- But, on the mathematical view, both of these implications are equally valid.

- "Berta had fewer than 3 drinks"
  => |B & D| < 3 => |B & D| < 4
  => "Berta had fewer than 4 drinks"

- "Berta had at most 2 drinks"
  => |B & D| ≤ 2 => |B & D| ≤ 3
  => "Berta had at most 3 drinks"

Methodological concerns

- Arguably not a pragmatically motivated task

- Given that A and B are uttered, whether or not A => B isn't relevant

- If A is uttered and A => B holds, it shouldn't be the case that B is uttered
  - unless the speaker wishes implicitly to deny A

- So given that A and B are uttered, assessing whether A => B doesn't seem to be a natural linguistic activity

Compatibility of sentences

(a) Berta had at most 2 drinks

(b) Berta had at most 3 drinks

G&N:

(a) requires 2 to be a possibility, but explicitly forbids 3

(b) requires 3 to be a possibility

(a) and (b) contradictory in this respect.

Compatibility judgements

Q. Is it the case that these two sentences contradict one another, or could they both be true?

Berta had at most 2 drinks

Berta had at most 3 drinks

15 participants, 3 items per condition (8 conditions)

At most 2 & At most 3: 53% compatible

Fewer than 3 & Fewer than 4: 84% compatible

Conditional reasoning

Q. Someone says "If Berta has had at most 3 drinks, she is fit to drive. Berta has had at most 2 drinks". *Does that person think that Berta is fit to drive?*
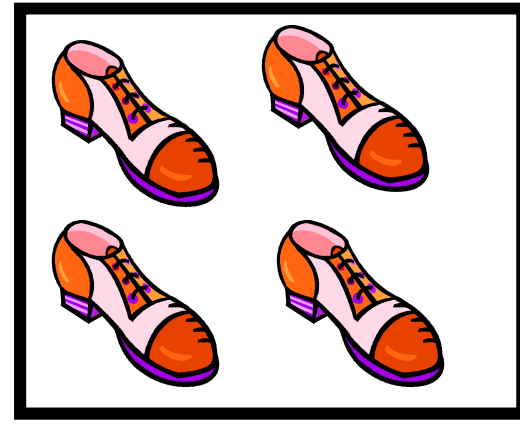
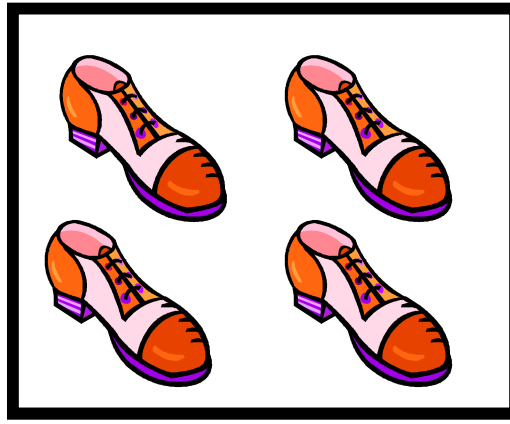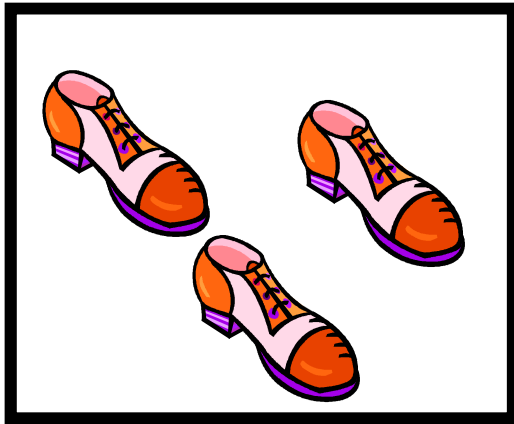*8 participants, 3 items (1 item for controls)*

At most 2 & At most 3: 96% acceptance

Fewer than 3 & Fewer than 4: 100% acceptance

Conditional reasoning (b)

Q. Someone says "Anyone who has had at most 3 drinks is fit to drive. Berta has had at most 2 drinks". *Does that person think that Berta is fit to drive?*

"At most" in set membership contexts

Summary

Based on these experimental findings

- either the core meaning is not modal in either case

- or the modal meaning disappears under compatibility judgement, conditional reasoning, or set membership contexts.
  - Why should this happen?

Outline of alternative account

- Quantifiers do not have core modal meaning

- Quantifier choice governed by combination of

  - Quantifier complexity / markedness
  - Informativeness
  - Salience or relevance of number being used
  - Other factors?

Example (1)

Scenario: There are 21-24 people in the room

Possible utterances:

- "There are more than 20 people…"
- "There are at least 21 people…"
- "There are at least 20 people…"

Example (1)

Scenario: There are 21-24 people in the room

Possible utterances:

- "There are more than 20 people…"
- "There are at least 21 people…"
- "There are at least 20 people…" **

Example (1)

Scenario: There are 21-24 people in the room

Possible utterances:

- "There are more than 20 people…"
- "There are at least 21 people…" **
- "There are at least 20 people…" **

Example (1)

Scenario: There are 21-24 people in the room

Possible utterances:

- "There are more than 20 people…" ✓
- "There are at least 21 people…" **
- "There are at least 20 people…" **

Example (2)

Scenario: There are 20-24 people in the room

Possible utterances:

- "There are at least 20 people…" *
- "There are more than 19 people…" *
- "There are at least 19 people…" ***

Generalisation

- "More than" is preferred both in cases where the lower bound is not a round number and in some cases where it is.

- "At least" is preferred only in some cases where the lower bound is a round number, in which cases it competes with "more than".

- Same goes for "less/fewer than" vs. "at most" (complicated by "not more than" etc.)

Relevance of number line

- If all numbers are equally salient/relevant, then there's no reason ever to choose "at least N" on the above account: "more than $N-1$" harmonically bounds it.

- But they are not (Dehaene 1997, Butterworth 1999, Jansen and Pollmann 2001, i.a.)

- The need to use a salient number might force your choice of quantifier (so no redundancy).

Quantifiers plus numbers

- British National Corpus:
  - "At least 20" > "More than 19" by 110 to 6
  - "More than 20" > "At least 21" by 357 to 23
  - Same goes for round numbers flanked by non-round numbers in general

Contradiction?

If the failure of the original inference ("at most two" implies "at most three") is not down to the core semantics, it must be some kind of pragmatic intrusion. Where does this come from?

Idea: "At most" without a licensing context gives rise to modal implicature => 'classical' inference fails.

Prediction: Use "at most" in contexts where it is licensed => higher acceptance rates of the 'classical' inference (pilot data).

Conclusion?

- Nothing so definitive!

- Geurts and Nouwen right to distinguish comparative and superlative quantifiers, but maybe not about superlative quantifiers having "core" modal meaning

- Much more work required

- Aim: show that the meaning difference in this case "drops out" of a full account of quantifier meaning based on something like constraints

Thank you!

Questions?

Contact:

Chris Cummins, crc37@cam.ac.uk

Napoleon Katsos, nk248@cam.ac.uk

Acceptance of non-modal "at most"?

- Could be argued that this is felicitously modal in the G&N sense

- However, slippery slope:

[?]"There are possibly four shoes in each box"

[??]"It is possible that there are four shoes in each box"

[???]"In each box, it is possible that there are four shoes"

Logical relatedness (2)

"Dave has two suits but Richard has at most one suit".

True or false: Dave and Richard each have at most two suits.

"At most" now placed in a more natural context in both the antecedent and consequent.

Logical relatedness (3)

"Dave has two suits but Richard has at most one suit".

True or false: Dave and Richard each have at most two suits.

27 participants, 3 items

Acceptance rate 43%.

Is the reasoning "at most 1" to "at most 2" embedded in this?